



Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS UNITS 3 AND 4

Section One:
Calculator-free

If required by your examination administrator, please
place your student identification label in this box

Student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (53 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(4 marks)

Determine the following:

(a) $\int 24(3x + 2)^3 dx.$

(2 marks)

(b) $\frac{d}{dx} \sin(3x + 2).$

(1 mark)

(c) $\frac{d}{dx} \int_2^x (3t + 2) dt.$

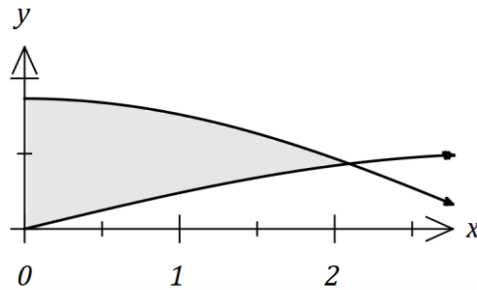
(1 mark)

Question 2

(6 marks)

Let $f(x) = \sqrt{3} \cos\left(\frac{x}{2}\right)$ and $g(x) = \sin\left(\frac{x}{2}\right)$.

The shaded region on the graph below is enclosed by $x = 0$, $y = f(x)$ and $y = g(x)$.



(a) Show that $f\left(\frac{2\pi}{3}\right) = g\left(\frac{2\pi}{3}\right)$.

(2 marks)

(b) Determine the area of the shaded region.

(4 marks)

Question 3

(7 marks)

(a) Write $\log_6 5 + 2 \log_6 2 - 1$ in the form $\log_6 k$.

(3 marks)

(b) Solve for x the equation $e^{3+x} = \sqrt{5}$.

(2 marks)

(c) Determine $\frac{d}{dx} \left(\ln \left(\frac{1}{1 + 2x^4} \right) \right)$.

(2 marks)

Question 4

(7 marks)

The velocity of a small body moving in a straight line at time t seconds is given by

$$v = \frac{8}{1+t} \text{ m/s}, \quad t \geq 0.$$

- (a) Determine the velocity of the body when its acceleration is -2 m/s^2 . (4 marks)

- (b) Calculate the distance travelled by the body in the first 3 seconds. (3 marks)

Question 5

(7 marks)

The random variable X has probability density function $f(x)$ shown below, where k is a positive constant.

$$f(x) = \begin{cases} kx + \frac{1}{12} & 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Deduce that $k = \frac{1}{6}$.

(3 marks)

(b) Determine the value of a if $P(a < X < 2) = \frac{1}{3}$.

(4 marks)

Question 6

(8 marks)

Let $f(x) = (1 + x)e^{-3x}$.

- (a) Determine the coordinates of the stationary point of the graph of $y = f(x)$ and use the second derivative test to determine its nature. (6 marks)

- (b) Determine the coordinates of the point of inflection of the graph of $y = f(x)$. (2 marks)

Question 7

(7 marks)

Let $f(x) = \frac{x}{x+1}$.

(a) Determine $f(x)$ and $f(x + \delta x)$ when $x = 70$ and $\delta x = 5$. (2 marks)

(b) Use $f(x)$ and the increments formula to estimate the difference between $\frac{89}{90}$ and $\frac{92}{93}$. (5 marks)

Question 8

(7 marks)

In a class of 16 students, 11 are right-handed.

- (a) One student is selected at random from the class and the random variable X is the number of right-handed students in the selection. Determine the mean and standard deviation of X . (3 marks)

- (b) Two students are selected at random from the class without replacement and the random variable Y is the number of right-handed students in the selection.

- (i) Complete the probability distribution table below. (3 marks)

y	0	1	2
$P(Y = y)$			

- (ii) Determine $E(Y)$. (1 mark)

Supplementary page

Question number: _____

